CONDENSATION OF SATURATED VAPOR ON A SPHERICAL DROPLET UNDER NEAR-CRITICAL CONDITIONS

B. I. Brounshtein, V. B. Brounshtein,
V. Ya. Rivkind, and I. N. Sventitskaya

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Condensation (evaporation) of droplets in an atmosphere of saturated vapor is analyzed. The equation of transient heat conduction with a variable droplet radius is solved on a computer. Universal graphs are plotted for calculating both the heating and the condensation rate.

In many practical situations the evaporation or the condensation of droplets occurs in an atmosphere of saturated vapor whose partial pressure is equal to unity. We will consider the case where the droplet radius is much larger than the free path of molecules. Supersaturation, which is possible during condensation, will be disregarded.

Without detracting from generality, we will analyze the condensation of droplets, assuming the initial droplet temperature T_0 to be lower than the vapor saturation temperature T_V. Under these assumptions, the vapor temperature may be considered constant down to the droplet surface, because a temperature drop at the surface would produce a corresponding vapor-pressure drop with a high rate of convective heat and mass transfer, which in turn would tend to equalize both the pressure and the temperature. Accordingly, the droplet buildup during condensation is determined from the boundary conditions at its dilating surface:

$$\lambda\left(\frac{\partial T}{\partial r}\right)_{r=R(t)}=\gamma_L\,(i_V-i_L)\,\frac{dR(t)}{dt}\,. \qquad (1)$$

Henceforth we will consider only droplets with small diameters in a low-velocity stream, for which $Pe < 1$. Convective heat transfer inside a droplet is negligible at a low Peclet number [1, 2]. In this case the problem of determining the condensation rate of saturated vapors reduces to solving the equation of transient heat conduction

$$\frac{\partial T}{\partial t}=\frac{a}{r^2}\cdot\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) \qquad (2)$$

with the boundary conditions (1) and

$$(T)_{r=0}\neq\infty;\ (T)_{r=R(t)}=T_V \qquad (3)$$

and the initial condition

$$(R)_{t=0}=R_0;\ (T)_{t=0}=T_0. \qquad (4)$$

We will now introduce the dimensionless variables

$$P=\frac{R}{R_0};\quad \rho=\frac{r}{R_0};\quad \theta=\frac{T-T_0}{T_V-T_0};\quad \tau=\frac{at}{R_0^2}\,. \qquad (5)$$

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Here

$$r = R_0\rho, \quad R = PR_0,$$

$$\frac{\partial}{\partial r} = \frac{1}{R_0} \cdot \frac{\partial}{\partial \rho}, \quad \frac{\partial T}{\partial t} = \frac{a}{R_0^2}(T_V - T_0)\frac{\partial \theta}{\partial \tau}$$

and (1)-(4) can be rewritten as

$$\frac{\partial \theta}{\partial \tau} = \frac{1}{\rho^2} \cdot \frac{\partial}{\partial \rho}\left(\rho^2 \frac{\partial \theta}{\partial \rho}\right), \tag{2a}$$

$$(\theta)_{\rho=0} \neq \infty, \quad (\theta)_{\rho=P(\tau)} = 1, \tag{3a}$$

$$\left(\frac{\partial \theta}{\partial \rho}\right)_{\rho=P(\tau)} = K\frac{dP(\tau)}{d\tau}\left(K = \frac{i_V - i_L}{c_v(T_V - T_0)}\right), \tag{1a}$$

$$(P)_{\tau=0} = 1, \quad (\theta)_{\tau=0} = 0 \quad (\rho \leqslant P(\tau)). \tag{4a}$$

We next find the maximum size of a droplet. The principal condition (1) will be replaced by an equivalent averaged equation of heat balance

$$\frac{4}{3}\pi R^3 \gamma_L c_v d\overline{T} = 4\pi R^2 \gamma_L \left[i_V - i_L + c_v(T_V - \overline{T})\right] dR \tag{6}$$

or, in dimensionless variables (5),

$$\frac{d\overline{\theta}}{3(1 + K - \overline{\theta})} = \frac{dP(\tau)}{P(\tau)}. \tag{7}$$

With $P = 1$ and $\overline{\theta} = 0$, integrating (7) yields

$$P(\tau) = \left(\frac{1 + K}{1 + K - \overline{\theta}}\right)^{\frac{1}{3}}. \tag{8}$$

Since $\overline{\theta} \to 1$ when $\tau \to \infty$, hence the maximum value of $P(\tau)$ is

$$P_{\max} = \left(1 + \frac{1}{K}\right)^{\frac{1}{3}}. \tag{9}$$

According to (9), $P_{\max} - 1 \ll 1$ when $K \gg 1$.

In this case the dilation of the droplet surface may be disregarded and the solution of Eq. (2a) reduces to the conventional sphere heating problem with the boundary condition (3a) and $P(\tau) = 1$. Moreover, into Eq. (1a) for the condensation rate we insert the value of $\theta(\rho)$ obtained from the solution to Eq. (2a) with a stationary boundary.

When the heat of evaporation is high and the temperature difference $T_V - T_0$ is small, then $K \gg 1$. At a temperature of saturated vapor near the critical point, however, K can be of the order of unity or less than unity and the droplet buildup must be taken into account in solving the equation of heat conduction.

The problem was solved numerically by the nonlinear variational difference scheme [3]:

$$\left[\rho_{i+\frac{1}{2}}\left(\theta_i^{j+1}\right)_{\hat{\rho}}\right]_{\underline{\rho}} = \left(\theta_i^{j+1}\right)_{\underline{\tau}}\rho_i^2 \quad (1 \leqslant i \leqslant n_{j+1} - 1, \ j = 0, 1, \ldots,), \tag{10}$$

$$\left(\theta_{n_{j+1}}^{j+1}\right)_{\underline{\rho}} \to K P_{\underline{\tau}}^{j+1}, \tag{11}$$

$$\theta_{n_{j+1}} = 1, \quad \theta_i^0 = 0, \quad P_0 = 1, \tag{12}$$

with $\rho_{i+1/2}$ denoting the value of ρ at point $i + 1/2$, with $\rho_{1/2} = 0$, and $v_{i\hat{\rho}}^{j+1}$, $v_{i\underline{\rho}}^{j+1}$, $v_{i\underline{\tau}}^{j+1}$ denoting the difference ratios of function v: central with respect to ρ at point $i + 1/2$, backward with respect to ρ at point i, and backward with respect to τ at point $\tau_{j+1} = \sum_{m=1}^{j+1} (\Delta\tau)_m$.

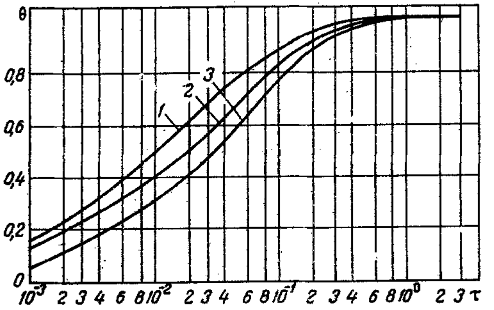

Fig. 1. Dimensionless temperature $\bar{\theta}$ as a function of τ: 1) K = 0.1; 2) 0.5; 3) 1-100.

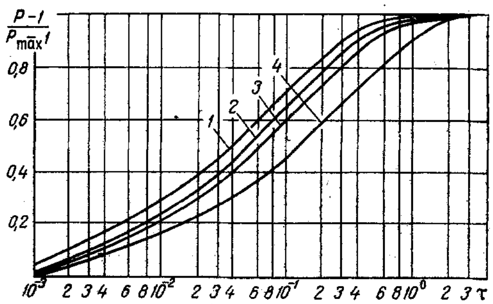

Fig. 2. Ratio $(P(\tau)-1)/(P_{max}-1)$ as a function of τ: 1) K = 2-200; 2) 1; 3) 0.5; 4) 0.1.

One can prove, on the basis of (3)-(7), that the approximate solution (10)-(12) converges toward the exact solution and thus

$$\operatorname{Sup}\int_0^{R(\tau)} \rho^2\left[\theta(\rho,\ \tau)-\theta_h'(\rho,\ \tau)\right]^2 d\rho+\int_0^T\int_0^{R(\tau)}\rho^2\left(\frac{\partial\left[\theta(\rho,\ \tau)-\theta_h'(\rho,\ \tau)\right]}{\partial\rho}\right)^2 d\rho d\tau\leqslant c_1(h^2+\tau),\quad T<\infty,$$

where c_1 is determined by K and T only while θ_h' is the polylinear complement of θ_h [3]. Equations (10)-(12) were solved by the method of successive approximations with an appropriate choice of h_i and $(\Delta\tau)_{j+1}$ steps. The calculation was carried to an asymptotic value $P(\infty)$ accurate to the fourth decimal.

The problem can also be solved by the method shown in [8] or by certain modifications of it. As can be demonstrated theoretically and as has been shown by trial computer calculations, however, this method is cumbersome because of the many operations required – more than in the grid method.

The results of computations are shown in Figs. 1 and 2. In Fig. 2 is shown the dimensionless mean temperature $\bar{\theta}$ as a function of the Fourier number τ at various values of K. In Fig. 2 is shown the corresponding relative change in the dimensionless droplet radius $(P(\tau)-1)/(P_{max}-1)$ as a function of τ, with P_{max} determined according to Eq. (9).

It follows from these graphs that, for solving the equation of heat conduction, dilation of the droplet surface may be disregarded when $K > 2$.

When $K < 1$, the condensation mode begins to deviate considerably from the quasisteady state. Numerical calculations were made for values of K down to 0.1. Such small values of K are attained in the case of ordinary liquids at vapor temperatures near the critical point. We note that to the limiting transition $T_V \to T_{cr}$ corresponds $K \to 0$ and, according to (9), a maximum increase in the droplet volume $P_{max} \to \infty$. For very small values of K, however, the assumptions made in this analysis merit further scrutiny.

NOTATION

T	is the temperature;
r	is the radial coordinate in the droplet;
R	is the droplet radius;
γ	is the density;
i	is the enthalpy;
a	is the thermal diffusivity;
c_V	is the specific heat of the liquid;
$K = (i_V - i_L)/c_V(T_V - T_0)$;	
$\theta = (T - T_0)/(T_V - T_0)$;	
$\tau = at/R_0^2$	is the Fourier number;
$P = R/R_0$;	
$\rho = r/R_0$.	

Subscripts

0	denotes the initial value;
L	denotes the liquid;
V	denotes the vapor.

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